

Meta-Mathematical Musings

Petra Cini and Raf Bocklandt

25 March 2023

Petra Cini - A brief introduction to my musical theory

My musical theory

Aim: to give a meta-reformulation of mathematical groups: a mapping from the world of mathematics to the one of music, using the sensorial concreteness of the latter.

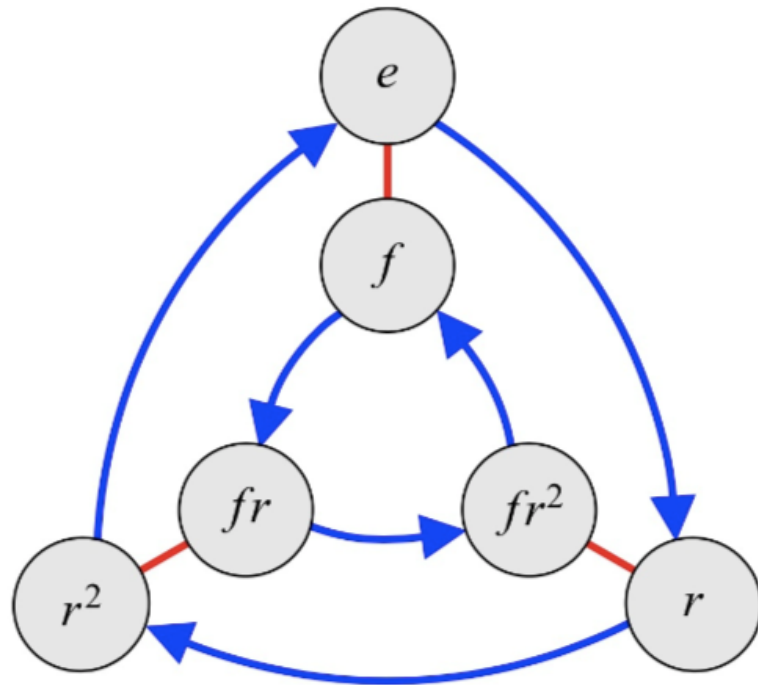
Key idea: analogical and perceptual analysis of the structure and elements of mathematical groups in terms of violence and purity.

Story of the idea

- 1) Benson, D. J. (2007). *Music: A Mathematical Offering*. Cambridge University Press
- 2) Carter, N. C. (2009). *Visual Group Theory*. The Mathematical Association of America

In Visual Group Theory group elements are presented as actions

Dihedral groups: generated by flips and rotations



my idea, sensation:

- flip (reflective symmetry)- violence
- rotation (rotational symmetry) - purity

Petra Cini - An introduction to my musical theory “A Damaged Purity: technique and theory of my musical practice” (2020)

- rhythm and counterpoint
- forms created as walks within groups analyzed in terms of violence and purity

On my website: <http://www.petracini.it/works/a-damaged-purity/>

A method, a way of living, a way of experiencing reality

Selected works concerning discrete symmetry

Petra Cini - ETUDES (2020-)

wallpaper groups

On Youtube: Petra Cini - Etude No.4 (2021-2022)

<https://www.youtube.com/watch?v=FpBjsB2L7HY>

Petra Cini - D3 <Gentleman, Jack> (2021)

dihedral group D3

On Youtube: Petra Cini - D3 < Gentleman, Jack> (2021)

<https://www.youtube.com/watch?v=MNpTWS3-e6s\>

From discrete to continuous

Question: What would happen if I were to apply my compositional ideas to continuous groups, in particular to Lie groups?

- to develop these ideas in the context of **continuous symmetry** I needed help, the expertise of someone that truly understood Lie groups, their essence.
- **why?** The focus is on musically representing mathematical objects, giving musical life to their meaning and to the sensations that can be found in them. It's not on their direct application or translation. In order to do this a true understanding of the mathematical structures in question is necessary.
- Eric Opdam, Raf Bocklandt

Math, Art and Music

- Mathematician at the University of Amsterdam, who specialises in Algebra and Geometry.
- Like to do outreach (e.g. talk at Universiteit van Nederland)
- Interest in visualisation of abstract concepts and the interplay with art
- Open for new projects to broaden my horizon and change my perspective

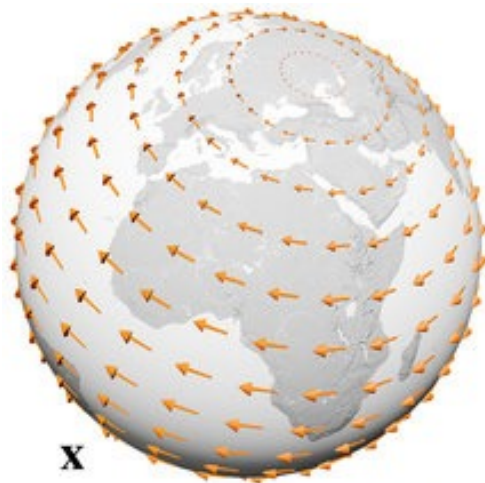
Musical representation of the Lie group $SO(3)$ = The $SO(3)$ Etudes

A musical representation of the Lie group $SO(3)$ generated by analyzing its structure and elements in terms of violence and purity

excerpt 1: the generators of $so(3)$

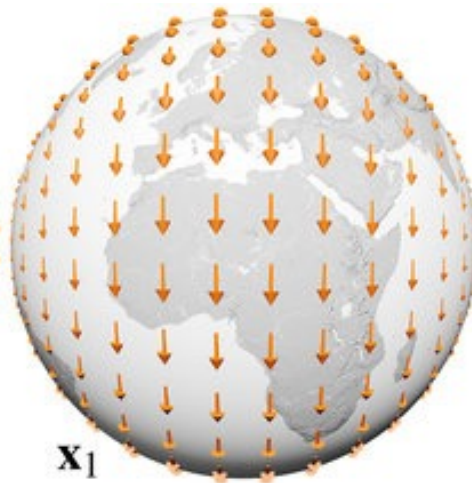
purity

- The presentation of the three dimensions of the Lie algebra $so(3)$
- The presentation, one after the other, of the generators of $so(3)$, i.e. the generators of rotations



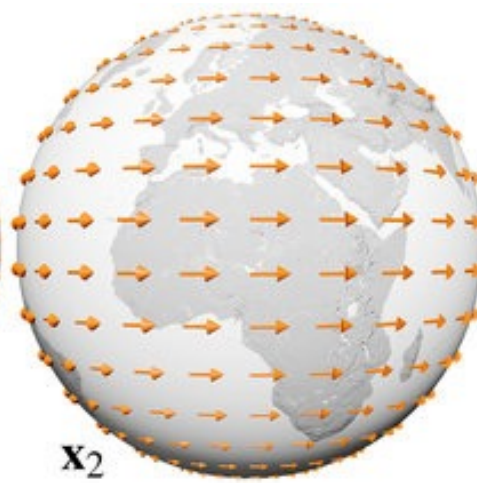
X

$$X = \begin{pmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{pmatrix}$$



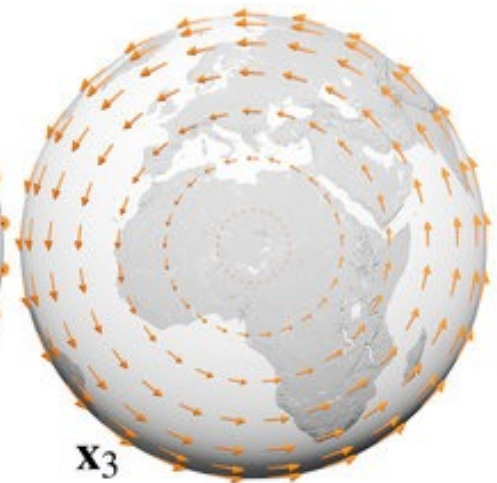
X₁

$$X_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$



X₂

$$X_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$



X₃

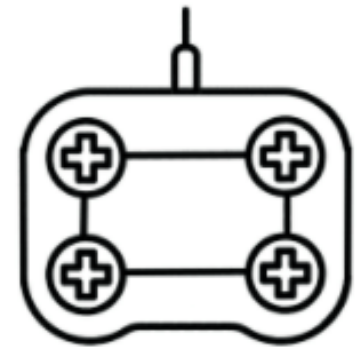
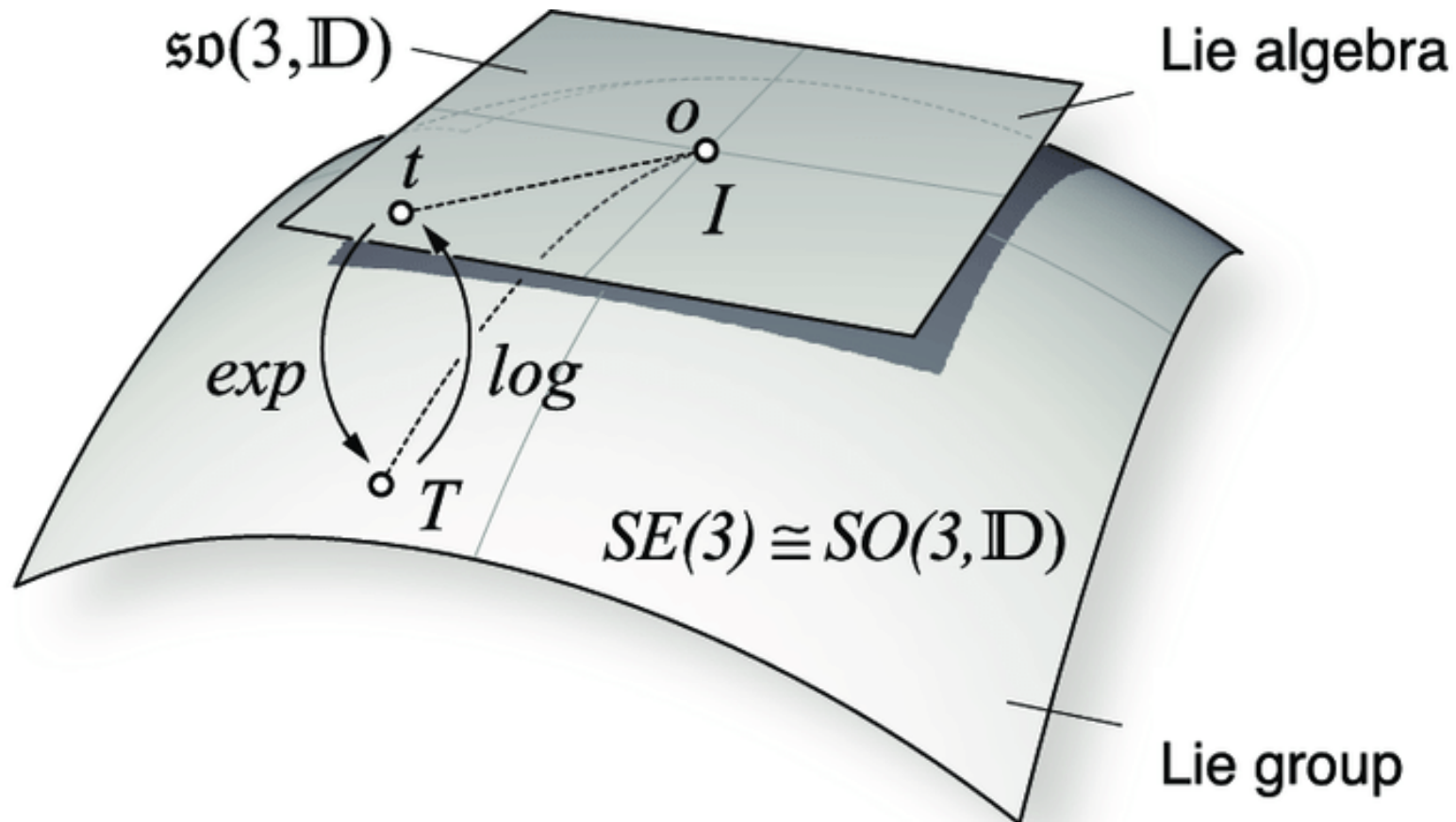
$$X_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

excerpt 2: the exponential map

purity \rightarrow violence

- from the Lie algebra $\mathfrak{so}(3)$ to the elements of the Lie group $SO(3)$

$z \rightarrow Rz$



excerpt 3: the walk within the group SO(3)

purity

Rx(30) - rotation of 30 degrees around the x axis - change in dynamics

Ry(30) - rotation of 30 degrees around the y axis - change in velocity

Rz(60) - rotation of 60 degrees around the x axis - change in pitch

A musical score for piano, consisting of two staves (treble and bass clef). The score is divided into several measures. The first measure has a trill (tr) over a note. The second measure is marked 'arpeggiando' and features a series of chords. The third measure has a trill (tr) over a note. The fourth measure has a trill (tr) over a note. The fifth measure has a trill (tr) over a note. The sixth measure has a trill (tr) over a note. The seventh measure has a trill (tr) over a note. The eighth measure has a trill (tr) over a note. The ninth measure has a trill (tr) over a note. The tenth measure has a trill (tr) over a note. The eleventh measure has a trill (tr) over a note. The twelfth measure has a trill (tr) over a note. The thirteenth measure has a trill (tr) over a note. The fourteenth measure has a trill (tr) over a note. The fifteenth measure has a trill (tr) over a note. The sixteenth measure has a trill (tr) over a note. The score includes various musical notations such as trills, arpeggiando, and dynamic markings.

Further work

Petra Cini

Youtube: <https://www.youtube.com/@PetraCini>

Website: <http://www.petracini.it/>

Theory: <http://www.petracini.it/works/a-damaged-purity/>

Raf Bocklandt

Youtube: <https://www.youtube.com/watch?v=1zVAPVUHrF0&t=191s>

Website: <http://algebra.hopto.org/wis/website/>

KdVI website: <https://kdvi.uva.nl/?cb>